

# Performance Analysis of Two-Step Bi-Directional Relaying with Multiple Antennas

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## Abstract

In this paper we study decode-and-forward multi-antenna relay systems that achieve bi-directional communication in two time slots. We investigate different downlink broadcast schemes which employ binary or analog network coding at the relay. We also analyze and compare their performances in terms of diversity order and symbol error probability. It is shown that if exact downlink channel state information is available at the relay, using analog network coding in the form of multi-antenna maximal-ratio transmit beamforming to precode the information vectors at the relay gives the best performance. Then, we propose a Max-Min antenna selection with binary network coding scheme that can approach this performance with only partial channel state information.

## I. INTRODUCTION

**B**I-DIRECTIONAL communications via a relay is often encountered in intra-cell, intra-hotspot, or more recently intra-picocell communication.

Traditionally, bi-directional relaying requires 4 channel uses (or 4 steps) as communication in each direction requires 2 channel uses, as shown in Fig. 1. However, the efficiency of the scheme can be improved by introducing network coding [1] at the relay node. The relay node may perform either analog network coding (ANC) [2]–[4] or binary network coding (BNC) [5], [6], while the destination nodes perform self interference cancellation, to reduce the channel uses of bi-directional relaying to 3 channel uses (or 3 steps). In this paper, we consider a 2-step approach by using two or more antennas at the relay, so that the relay node is able to jointly detect both messages sent from the two source nodes. The two messages are then combined using either binary eXclusive-OR (XOR) [2], [7] or analog beamforming. This two-step procedure is illustrated in Fig. 2.

In this paper, we study two different relaying schemes, ANC-based Transmit Beamforming (TB) and BNC-based Space Time Block Coding (STBC-BNC), and propose a novel BNC-based Max-Min Antenna Selection (Max-Min AS-BNC) scheme. In the Max-Min AS-BNC scheme, we select an antenna that has

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the largest channel gain among the worst channels from each antenna to either nodes A or B. We analyze the diversity gain and symbol error probability for the various multi-antenna relaying schemes to get an insight into their relative system performance and to compare their implementation requirements such as the need for downlink ( $2^{nd}$  step in Fig. 2) channel state information (CSI), *i.e.* phase and amplitude of the channels between each relay antennas and each node (A and B). It is shown that Max-Min AS-BNC achieves full diversity order with the need of coarse downlink CSI, *i.e.* the relative amplitude of the downlink channel, and approaches the performance of Transmit Beamforming (TB) based on maximal ratio AF transmission which requires exact CSI, and performs better than Space Time Block Coding with Binary Network Coding (STBC-BNC) which requires no CSI.

In this paper, bold lower case and upper case letters denote vectors and matrices, respectively;  $[\cdot]^T$  the transpose of a vector or a matrix;  $|\cdot|$ , the absolute value of a real number;  $\|\cdot\|$ , the norm of a vector and  $Q(x)$ , the  $Q$ -function of  $x$  which is the probability that a standard normal random variable will obtain a value larger than  $x$ .

The paper is organized as follows. In the next section, we present the system model for the different two-step two-way relaying schemes. In Section III, we analyze the downlink channel for different relaying schemes, in terms of symbol error probability and diversity order. We present the numerical results and compare them with our analytical results in Section IV. We then conclude our paper in Section V.

## II. SYSTEM MODEL

We consider a network comprising of a pair of single-antenna nodes (nodes A and B that would like to exchange information) and one  $N$ -antenna relay node (node R). All nodes are half-duplex and there is no direct link between nodes A and B. In the first transmission interval, nodes A and B transmit  $s_A$  and  $s_B$  respectively to the relay R simultaneously and on the same frequency. The multi-antenna relay R then uses maximum likelihood (ML) detection to detect them jointly.

We assume that the probability distribution of all the uplink channels and the downlink channels are the same, and the noise at both nodes A and B is complex zero-mean additive white Gaussian with variance  $\sigma_S^2$ . We will analyze the performance of the system for the information flow from node B to node A, where the results hold for the information flow from node A to node B by assuming that the channels have the same distribution statistic.

In the first time slot (uplink), the relay R receives

$$\mathbf{y}_R = \mathbf{H}_{up}\mathbf{s} + \mathbf{n}_R \quad (1)$$

where  $\mathbf{s} = (s_A \ s_B)^T$ ,  $\mathbf{h}_{AR} = (h_{A1} \dots h_{AN})^T$ ,  $\mathbf{h}_{BR} = (h_{B1} \dots h_{BN})^T$ ,  $\mathbf{H}_{up} = (\mathbf{h}_{AR} \ \mathbf{h}_{BR})$ , and  $\mathbf{n}_R$  is a circularly symmetric complex Gaussian noise vector  $\sim \mathcal{CN}(\mathbf{0}_N, \sigma_R^2 \mathbf{I}_N)$ .  $h_{kj}$  is a Rayleigh flat fading

channel with power mean  $E[|h_{kj}|^2] = \sigma_0^2$  between node  $k$  and the  $j^{th}$  relay antenna where  $k = A$  or  $B$  and  $j \in \{1, \dots, N\}$ .

We assume that  $\mathbf{H}_{up}$  is known at the relay. So, the relay performs maximum-likelihood decoding on the received vector to obtain

$$\hat{\mathbf{s}} = (\hat{s}_A \ \hat{s}_B)^T = \arg \min_{\mathbf{s}} \|\mathbf{y}_R - \mathbf{H}_{up}\mathbf{s}\|^2 \quad (2)$$

where  $\hat{s}_A$  and  $\hat{s}_B$  are the estimation of the transmitted symbols by nodes  $A$  and  $B$ .

In the second time slot (downlink), the relay uses different multi-antenna precoding schemes to broadcast a message, that is formed by  $\hat{s}_A$  and  $\hat{s}_B$ , to both nodes  $A$  and  $B$ . If the downlink scheme is based on binary network coding (BNC), the relay performs bit-wise XOR operation on the estimated symbols  $\hat{s}_A$  and  $\hat{s}_B$  to obtain the network coded message

$$s_{XOR} = \hat{s}_A \oplus \hat{s}_B.$$

Then, the relay broadcasts  $\mathbf{s}_R = f(s_{XOR})$ , which is a function of  $s_{XOR}$ . If the downlink scheme is based on analog network coding (ANC),  $\mathbf{s}_R$  is a linear combination of the symbols  $\hat{s}_A$  and  $\hat{s}_B$ ,  $\mathbf{s}_R = f(\hat{s}_A, \hat{s}_B)$ . In both cases,  $\mathbf{s}_R$  is broadcasted by the relay to both nodes  $A$  and  $B$  in the second time slot, as shown in Fig. 2.

We denote the downlink channel vectors from the relay node to nodes  $A$  and  $B$  by  $\mathbf{h}_{RA} = (h_{1A} \dots h_{NA})^T$  and  $\mathbf{h}_{RB} = (h_{1B} \dots h_{NB})^T$  respectively. Without loss of generality,  $h_{jk}$  is a Rayleigh flat fading channel with the same mean power as the uplink channels, *i.e.*  $E[|h_{jk}|^2] = \sigma_0^2$  between the  $j^{th}$  relay antenna and node  $k$  where  $k = A$  or  $B$  and  $j \in \{1, \dots, N\}$ . The average SNR at the relay node and at node  $A$  or  $B$  are  $\zeta_R = \frac{\mathcal{P}}{\sigma_R^2}$  and  $\zeta_S = \frac{\mathcal{P}}{\sigma_S^2}$  respectively, where  $\mathcal{P} = E[|s_A|^2] = E[|s_B|^2] = E[|\mathbf{s}_R|^2]$ .

Node  $A$  receives

$$y_A = \mathbf{h}_{RA}^T \mathbf{s}_R + n_A \quad (3)$$

If the relaying scheme is based on BNC, after estimating  $s_{XOR}$ , node  $A$  performs bit-wise XOR operation on the estimated  $s_{XOR}$  and  $s_A$  to estimate  $s_B$ . If the relaying scheme is based on ANC, after canceling off the contribution of  $s_A$  from the received signal, node  $A$  estimates  $s_B$ .

We consider three different relaying schemes which require different amount of downlink channel state information (CSI) at the relay:

- No CSI: STBC-BNC relaying scheme,
- Coarse CSI: Antenna selection relaying scheme (*i.e.* Max-Min AS-BNC),
- Exact CSI: Transmit beamforming (TB) relaying scheme (based on ANC).

In the exact CSI case, the relay knows the amplitude and phase of the downlink channels. In the coarse CSI case, the relay just needs to know the relative amplitude of the downlink channels. Although CSI can be

inferred from the uplink in TDD system, this is usually not feasible due to non-symmetrical impairments in the transmit and receive circuitry. Therefore, the proposed Max-Min antenna selection that requires only coarse CSI is more practical than the transmit beamforming that requires perfect CSI.

We will show later that Max-Min antenna selection with BNC can achieve full diversity, and it approaches the performance of TB relaying scheme asymptotically. In addition to the fact that Max-Min AS-BNC just needs coarse CSI, another advantage of using Max-Min AS-BNC relaying scheme over TB is that in a distributed network of multiple-antenna relays, implementing Max-Min AS-BNC is much easier than implementing TB. To implement Max-Min AS-BNC, a method based on time is selected: each relay will start its own timer with an initial value, inversely proportional to  $g_k$ . The relay with the least initial value or the most  $g_k$  gets zero earlier than the other relays [8]. In this case to implement TB scheme we need much more complicated algorithm to select one or two relay(s) to transmit  $\mathbf{s}_R$  which results in the most instantaneous SNR at nodes A and B.

### III. DOWNLINK ANALYSIS

In the following, we derive the diversity order and symbol error probability of the different downlink channel schemes and hence of the whole system. We denote an exponential random variable  $D$  with parameter  $\lambda$  as one with  $E[D] = 1/\lambda$ ; a Gamma-distributed random variable  $T$  with parameters  $(\Omega, \theta)$  as one with  $E[T] = \Omega\theta$  and  $var(T) = \Omega\theta^2$ . Also, a random variable  $Z$  which is complex Gaussian-distributed with parameters  $(\mu, \sigma^2)$  has  $E[Z] = \mu$  and  $var(Z) = \sigma^2$ .

We assume both nodes A and B have sufficient downlink channel knowledge to perform decoding. We employ M-PSK constellation and coherent detection at the receiver, so according to [9]–[12], we can write the symbol error probability (SEP) for the downlink channels as

$$P_d = E_\gamma[P_{dA}] = 1/\pi \int_0^{\pi-\pi/M} \psi_\gamma(g_{MPSK}/\sin^2 \theta) d\theta, \quad (4)$$

where  $P_{dA}$  is the SEP of the downlink channels at node A,  $g_{MPSK} = \sin^2(\pi/M)$ ,  $\gamma$  is the instantaneous receive SNR and  $\psi_\gamma(t)$  is the moment generating function (MGF) of  $\gamma$ . Note that the proposed schemes can be applied to any modulation. We use the M-PSK to demonstrate how the analysis can be done, and the extension to other constellations is straightforward.

According to [10], the diversity order of the downlink channel can be found by

$$\lim_{\zeta_S \rightarrow \infty} -\frac{\log(\psi_\gamma(g_{MPSK}))}{\log(\zeta_S)}. \quad (5)$$

#### A. Max-Min Antenna Selection with Binary Network Coding (Max-Min AS-BNC)

In the Max-Min AS-BNC scheme, we select just one antenna of the relay node to send  $s_{XOR}$ , at all times. The process of selection is done in two steps. First, we select the worst downlink channel  $h_{ik_i}$  for

every antenna  $i \in \{1, \dots, N\}$ , *i.e.*

$$k_i = \arg \min_r |h_{ir}|, \quad r \in \{A, B\} \quad (6)$$

Then, the relay antenna  $j$  with the best downlink channel among those in (6) is selected, *i.e.*

$$j = \arg \max_i |h_{ik_i}|, \quad i \in \{1, \dots, N\}. \quad (7)$$

This antenna  $j$  at the relay node is then used to broadcast  $s_{XOR}$  to nodes A and B. So, node A receives

$$y_A = h_{jA}s_{XOR} + n_A. \quad (8)$$

Hence, the instantaneous SNR,  $\gamma$ , at node A is

$$\gamma = |h_{jA}|^2 \zeta_S. \quad (9)$$

In this relaying scheme, there are two states. State 1 is when the channel magnitude from the selected antenna at the relay node to node A is less than to node B, *i.e.*  $|h_{jA}| < |h_{jB}|$ , or

$$\min\{|h_{jA}|, |h_{jB}|\} = |h_{jA}|, \quad (10)$$

and State 2 is when the channel magnitude from the selected antenna at the relay node to node A is more than to node B, *i.e.*  $|h_{jA}| > |h_{jB}|$ , or

$$\min\{|h_{jA}|, |h_{jB}|\} = |h_{jB}|. \quad (11)$$

Hence, the SEP at node A is

$$\text{SEP} = \text{SEP}_1 P_1 + \text{SEP}_2 P_2, \quad (12)$$

where  $\text{SEP}_i$  is the SEP conditioned on State  $i$  and  $P_i$  is the probability of occurrence of State  $i$ . Since the channel gain in State 1 is less than the channel gain in State 2, the symbol error probability of State 1 will be higher than the symbol error probability of State 2, *i.e.*  $\text{SEP}_1 > \text{SEP}_2$ . Since all channels are i.i.d,  $P_1 = P_2 = 1/2$ , the overall SEP will be upper-bounded by  $\text{SEP}_1$  and lower-bounded by  $\text{SEP}_1/2$ , *i.e.*

$$\frac{\text{SEP}_1}{2} \leq \text{SEP} \leq \text{SEP}_1. \quad (13)$$

To use the MGF to work out the diversity order and SEP of the downlink channel, we need to know the PDF of  $|h_{jA}|^2$  in State 1.

From [13], the cumulative density function (CDF) of  $|h_{ik_i}|^2$  can be shown to be

$$F(|h_{ik_i}|^2) = 1 - \exp(-2|h_{ik_i}|^2/\sigma_0^2). \quad (14)$$

Hence, the PDF of  $|h_{jA}|^2$  in State 1 is

$$f(|h_{jA}|^2) = \frac{2N}{\sigma_0^2} \exp(-2|h_{jA}|^2/\sigma_0^2) (1 - \exp(-2|h_{jA}|^2/\sigma_0^2))^{N-1}. \quad (15)$$

Using binomial expansion, we have

$$(1 - \exp(-2|h_{jA}|^2/\sigma_0^2))^{N-1} = \sum_{k=0}^{N-1} \binom{N-1}{k} (-1)^k \exp(-2k|h_{jA}|^2/\sigma_0^2). \quad (16)$$

The MGF of the instantaneous SNR is thus

$$\begin{aligned} \psi_\gamma(t) &= E[\exp(-t\gamma)] \\ &= \int_0^\infty \exp(-t|h_{jA}|^2\zeta_S) f(|h_{jA}|^2) d|h_{jA}|^2. \end{aligned} \quad (17)$$

Substituting (16) into (15) and then using (17), the MGF of the instantaneous SNR in the Max-Min AS-BNC scheme in State 1 is

$$\begin{aligned} \psi_\gamma(t) &= \sum_{k=0}^{N-1} \binom{N-1}{k} \frac{2N(-1)^k}{\sigma_0^2} \int_0^\infty \exp(-|h_{jA}|^2(t\zeta_S + 2(k+1)/\sigma_0^2)) d|h_{jA}|^2 \\ &= \sum_{k=0}^{N-1} \binom{N-1}{k} \frac{N(-1)^k}{1 + k + t\sigma_0^2\zeta_S/2} \end{aligned} \quad (18)$$

$$= \frac{N!}{\prod_{k=0}^{N-1} (k+1 + t\sigma_0^2\zeta_S/2)}. \quad (19)$$

The equality between (18) and (19) can be easily shown using partial fraction decomposition [14].

According to (5), the diversity order of the downlink channel of the Max-Min AS-BNC scheme in State 1 is

$$\begin{aligned} G_d &= \lim_{\zeta_S \rightarrow \infty} -\frac{\log N!}{\log \zeta_S} + \lim_{\zeta_S \rightarrow \infty} \frac{\sum_{k=0}^{N-1} \log(k+1 + g_{MPSK}\sigma_0^2\zeta_S/2)}{\log \zeta_S} \\ &= 0 + N = N. \end{aligned} \quad (20)$$

Following the inequalities in (13), we conclude that the Max-Min AS-BNC relaying scheme is able to achieve the full diversity order of  $N$  for the downlink channel and also for the whole system.

Similarly, using (19), the SEP of the downlink channel can be lower-bounded as

$$P_d^{Max-Min} = E_h[P_{eA}] \geq \frac{1}{2\pi} \int_0^{\pi-\pi/M} \psi_\gamma\left(\frac{g_{MPSK}}{\sin^2 \theta}\right) d\theta. \quad (21)$$

### B. Transmit Beamforming (TB)

In the TB downlink relaying scheme, the detected uplink data  $\hat{s}_A$  and  $\hat{s}_B$  are precoded using analog transmission weight vectors  $\mathbf{v}_B$  and  $\mathbf{v}_A$  respectively based on the principles of maximum ratio transmission (MRT) [15]

$$\mathbf{v}_A = \frac{\mathbf{h}_{RA}^*}{\|\mathbf{h}_{RA}\|}, \mathbf{v}_B = \frac{\mathbf{h}_{RB}^*}{\|\mathbf{h}_{RB}\|}. \quad (22)$$

The relay R then sends

$$\mathbf{s}_R = \frac{\mathbf{v}_B \hat{s}_A + \mathbf{v}_A \hat{s}_B}{\sqrt{2}}. \quad (23)$$

where  $\sqrt{2}$  is to normalize the transmission power at the relay node. The receiver output at node A, after subtracting  $\mathbf{h}_{RA}^T \mathbf{v}_B \hat{s}_A$  from the received vector is

$$y_A = \frac{\mathbf{h}_{RA}^T \mathbf{v}_A \hat{s}_B}{\sqrt{2}} + n_A = \frac{\|\mathbf{h}_{RA}\| \hat{s}_B}{\sqrt{2}} + n_A. \quad (24)$$

The instantaneous SNR ( $\gamma$ ) at node A will be

$$\gamma = \frac{\|\mathbf{h}_{RA}\|^2 \zeta_S}{2}. \quad (25)$$

Since we assume perfect cancellation at the receiver, the downlink channel is equivalent to a point-to-point MISO channel. The optimal precoding scheme for point-to-point transmission is MRT. This explains our choice of (22) as the TB weight vectors. Since TB linearly sums  $\hat{s}_A$  and  $\hat{s}_B$ , it is a form of ANC (analog network coding). From [16], the diversity order of MRT is  $N$ , therefore the diversity order of the downlink channel and the entire bi-directional relaying system is  $N$ .

Since  $\|\mathbf{h}_{RA}\|^2 = \sum_{i=1}^N |h_{iA}|^2$  is the sum of  $N$  independent exponentially distributed random variables each with  $\lambda = 1/\sigma_0^2$ ,  $\|\mathbf{h}_{RA}\|^2$  is a Gamma random variable with parameters  $(N, \sigma_0^2)$ . Hence, the MGF of  $\gamma$  is

$$\psi_\gamma(t) = \left( \frac{\lambda}{\lambda + t\zeta_S/2} \right)^N = \left( \frac{1}{1 + t\sigma_0^2\zeta_S/2} \right)^N, \quad (26)$$

Using (26) and (4), the SEP can be written as

$$P_d^{TB} = E_h[P_{dA}] = \frac{1}{\pi} \int_0^{\pi-\pi/M} \psi_\gamma \left( \frac{g_{MPSK}}{\sin^2 \theta} \right) d\theta. \quad (27)$$

### C. Space-Time Block Coding with Binary Network Coding (STBC-BNC)

For the STBC-BNC scheme, at the relay node, binary network coded symbols at times  $n, \dots, n+N$  are buffered and encoded using STBC to produce  $\mathbf{s}_R$ . Alamouti coding is an orthogonal STBC for two transmit antennas, which achieves full diversity order without sacrificing transmission rate. In the Alamouti-BNC scheme, the downlink signal  $\mathbf{s}_R$  is

$$\mathbf{s}_R = \begin{pmatrix} s_{XOR}(n) & -s_{XOR}^*(n+1) \\ s_{XOR}(n+1) & s_{XOR}^*(n) \end{pmatrix}.$$

Correspondingly, node A receives

$$\begin{pmatrix} y_A(n) & y_A(n+1) \end{pmatrix} = \frac{\mathbf{h}_{RA}^T \mathbf{s}_R}{\sqrt{2}} + n_A. \quad (28)$$

In general, by using orthogonal STBC at an  $N$ -antenna relay, it is easy to show that the instantaneous SNR ( $\gamma$ ) received at the destination node is

$$\gamma = \frac{\|\mathbf{h}_{RA}\|^2 \zeta_S}{N}, \quad (29)$$

which is the same as (25) for the TB relaying scheme except for the denominator of  $N$  instead of 2. Hence, we conclude that the diversity order of the downlink channel and also the whole system, is  $N$  for the STBC-BNC relaying scheme.

For the same reason, the MGF of  $\gamma$  is the same as (26) with  $\zeta_S$  changed to  $\zeta_S/N$

$$\psi_\gamma(t) = \left( \frac{1/\sigma_0^2}{1/\sigma_0^2 + t\zeta_S/N} \right)^N. \quad (30)$$

Using (30) and (4), the SEP can be written as

$$P_d^{\text{STBC-BNC}} = E_h[P_{dA}] = \frac{1}{\pi} \int_0^{\pi-\pi/M} \psi_\gamma \left( \frac{g_{\text{MPSK}}}{\sin^2 \theta} \right) d\theta. \quad (31)$$

#### IV. SIMULATION AND DISCUSSION

##### A. Diversity and SEP Comparison

We approximate the overall performance by assuming that overall bi-directional communication will be erroneous if an error event occurs in either the uplink or the downlink. Hence,

$$P_{\text{tot}} \approx P_u + P_d(1 - P_u), \quad (32)$$

where  $P_u$ ,  $P_d$  and  $P_{\text{tot}}$  are SEP of the uplink channel, downlink channel and the whole system respectively. Since  $P_u P_d \ll 1$ , (32) can be simplified to

$$P_{\text{tot}} \approx P_u + P_d. \quad (33)$$

The union bound property [17] is used to compute an upper bound on  $P_u$ ,

$$P_u \leq \sum_{\mathbf{s} \neq \mathbf{s}_0} Q \left( \sqrt{\frac{\|(\mathbf{s}_0 - \mathbf{s})\mathbf{h}\|^2}{2\sigma_R^2}} \right), \quad (34)$$

while the SEP derived for the different downlink schemes in (21), (27) and (31) will be used as  $P_d$  in (33).

In Fig. 3, we plot the analytical and simulated SEP of the ANC and BNC-based relaying schemes. Close agreement between the analytical result and the simulation result, especially in the high SNR region, is



observed. Also, TB has an advantage of 0.5 dB over the BNC-based schemes. Interestingly, the Max-Min AS-BNC scheme and Alamouti-BNC scheme have very close performance at all SNRs.

In order to verify our analysis of the diversity order of Max-Min AS-BNC, we show the simulation results of the downlink portion with various antenna configuration in Fig. 4. From the figure, we can corroborate that the scheme achieves full downlink diversity order. Since the uplink performs ML decoding, we may conclude that the scheme achieves full end-to-end diversity order too. Additionally, we compare the Max-Min AS-BNC and STBC-BNC schemes by simulation for four-antenna relay in Fig. 5. It shows that the Max-Min AS-BNC scheme has a coding gain over the other. This advantage will be analyzed in the next section.

### B. Asymptotic Analysis

We analyze the asymptotic SEP ratio of downlink relaying scheme A relative to scheme B as:  $\frac{SEP_A}{SEP_B} \big|_{SNR \rightarrow \infty}$ . If this ratio is less than one, the relaying scheme A is deemed to have SEP performance gain over the relaying scheme B asymptotically.

We begin with the SEP ratio of TB over Max-Min AS-BNC

$$\begin{aligned}
 \frac{SEP_{TB}}{SEP_{Max-Min \text{ AS-BNC}}} \big|_{SNR \rightarrow \infty} &= \lim_{\zeta_S \rightarrow \infty} \frac{\int_0^{\frac{\pi}{M}} \frac{2 \sin^2 \theta}{(\sin^2 \theta + g_{MPSK} \sigma_0^2 \zeta_S / 2)^N} d\theta}{\int_0^{\frac{\pi}{M}} \frac{N! \sin^2 \theta}{\prod_{k=0}^{N-1} (\sin^2 \theta (k+1) + g_{MPSK} \sigma_0^2 \zeta_S / 2)} d\theta} \\
 &= \lim_{\zeta_S \rightarrow \infty} \frac{\int_0^{\frac{\pi}{M}} \frac{2 \sin^2 \theta}{(g_{MPSK} \sigma_0^2 \zeta_S / 2)^N} d\theta}{\int_0^{\frac{\pi}{M}} \frac{N! \sin^2 \theta}{\prod_{k=0}^{N-1} (g_{MPSK} \sigma_0^2 \zeta_S / 2)} d\theta} \\
 &= \lim_{\zeta_S \rightarrow \infty} \frac{(g_{MPSK} \sigma_0^2 \zeta_S / 2)^N \int_0^{\frac{\pi}{M}} 2 \sin^2 \theta d\theta}{(g_{MPSK} \sigma_0^2 \zeta_S / 2)^N N! \int_0^{\frac{\pi}{M}} \sin^2 \theta d\theta} \\
 &= \frac{2}{N!}.
 \end{aligned} \tag{35}$$

Next, the SEP ratio of STBC-BNC over Max-Min AS-BNC is

$$\begin{aligned}
 \frac{SEP_{STBC-BNC}}{SEP_{Max-Min \text{ AS-BNC}}} \big|_{SNR \rightarrow \infty} &= \lim_{\zeta_S \rightarrow \infty} \frac{\int_0^{\frac{\pi}{M}} \frac{2 \sin^2 \theta}{(\sin^2 \theta + g_{MPSK} \sigma_0^2 \zeta_S / N)^N} d\theta}{\int_0^{\frac{\pi}{M}} \frac{N! \sin^2 \theta}{\prod_{k=0}^{N-1} (\sin^2 \theta (k+1) + g_{MPSK} \sigma_0^2 \zeta_S / 2)} d\theta} \\
 &= \lim_{\zeta_S \rightarrow \infty} \frac{\int_0^{\frac{\pi}{M}} \frac{2 \sin^2 \theta}{(g_{MPSK} \sigma_0^2 \zeta_S / N)^N} d\theta}{\int_0^{\frac{\pi}{M}} \frac{N! \sin^2 \theta}{\prod_{k=0}^{N-1} (g_{MPSK} \sigma_0^2 \zeta_S / 2)} d\theta} \\
 &= \lim_{\zeta_S \rightarrow \infty} \frac{(g_{MPSK} \sigma_0^2 \zeta_S / N)^N \int_0^{\frac{\pi}{M}} 2 \sin^2 \theta d\theta}{(g_{MPSK} \sigma_0^2 \zeta_S / 2)^N N! \int_0^{\frac{\pi}{M}} \sin^2 \theta d\theta} \\
 &= \frac{N^N}{2^{N-1} N!}.
 \end{aligned} \tag{36}$$

(35) and (36) suggest that with two antennas at the relay, all the relaying schemes have the same downlink SEP performance, while with more than two antennas at the relay, TB achieves the best

downlink SEP performance, followed by Max-Min AS-BNC, then STBC-BNC. These analysis explain the observations in Fig. 3 and Fig. 5 respectively. Note that, although we use the lower bound of the SEP of Max-Min AS-BNC in the SEP ratio derivation, the simulation results in Fig. 3 have shown that the lower bound is very tight especially in the high SNR region.

## V. CONCLUSION

In this paper, we present and analyze three different multi-antenna relaying schemes with binary or analog network coding at the relay that achieve bi-directional wireless relay communication in two steps. The relaying scheme with analog network coding employs Transmit Beamforming (TB) with maximal ratio transmission. The relaying schemes with binary network coding employ Max-Min Antenna Selection (Max-Min AS-BNC) and Space-Time Block Coding (STBC-BNC). We derive the diversity order and symbol error probability formulas of these two-step bi-directional relaying schemes considering two single-antenna nodes connected via a multi-antenna relay.

We show that while all the relaying schemes presented achieve full diversity order, different schemes allow for different trade-offs between complexity and performance. Specifically, TB performs best in terms of SEP performance, but it needs exact downlink CSI at the relay. Max-Min AS-BNC comes next and is able to tolerate coarse CSI and can be implemented in a distributed multiple-relay system, followed by STBC-BNC which does not require CSI. Asymptotic SEP ratios are also derived to help evaluate the relative performance of these full-diversity two-way relaying schemes when the SNR is high.

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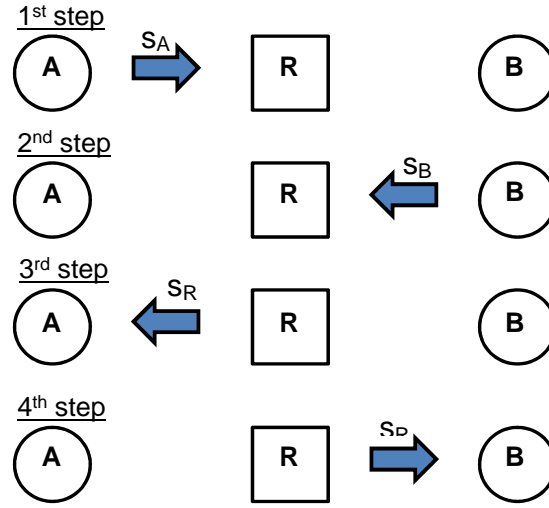


Fig. 1. Conventional four-step bi-directional relaying communication.

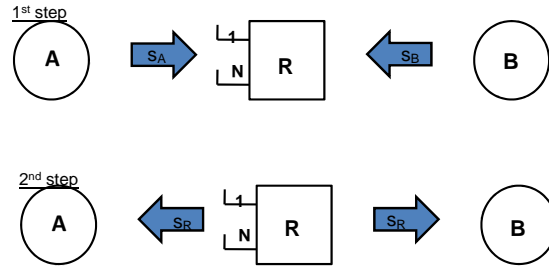


Fig. 2. Two-Step bi-directional communication with multi-antenna relay. Relay R broadcasts precoded versions of  $s_R = \hat{s}_A \oplus \hat{s}_B$  or linear combination of the symbols  $\hat{s}_A$  and  $\hat{s}_B$  to nodes A and B in the second time slot.

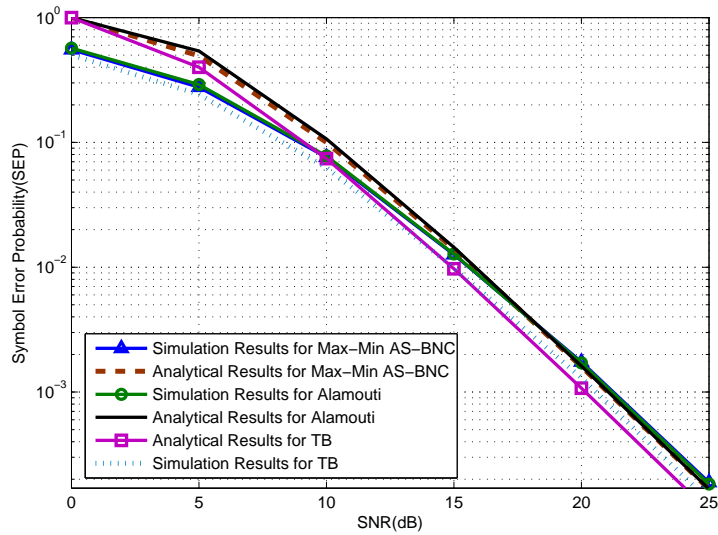


Fig. 3. Simulation and Analytical SEP of two-way relaying using QPSK with TB, Max-Min AS-BNC and Alamouti-BNC.

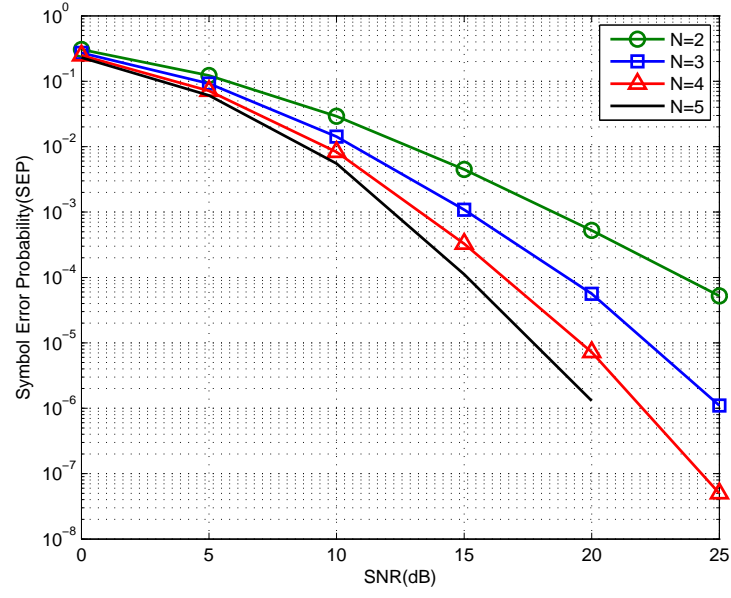


Fig. 4. Simulation results of downlink (2nd step) of Max-Min AS-BNC with different relay antenna number  $N$ .

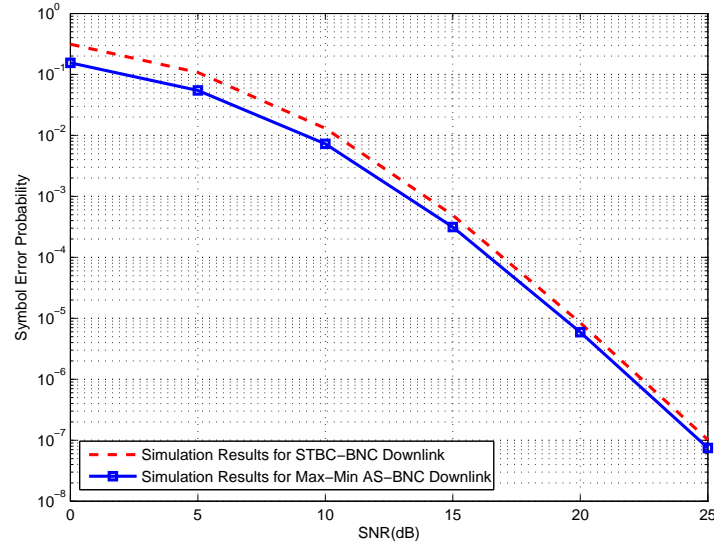


Fig. 5. Analytical SEP of two-way relaying using QPSK with Max-Min AS-BNC and STBC-BNC for four-antenna relay.

# Performance Analysis of Two-Step Bi-Directional Relaying with Multiple Antennas

Mahshad Eslamifar, Woon Hau Chin, Chau Yuen and Yong Liang Guan

## Abstract

In this paper we study decode-and-forward multi-antenna relay systems that achieve bi-directional communication in two time slots. We investigate different downlink broadcast schemes which employ binary or analog network coding at the relay. We also analyze and compare their performances in terms of diversity order and symbol error probability. It is shown that if exact downlink channel state information is available at the relay, using analog network coding in the form of multi-antenna maximal-ratio transmit beamforming to precode the information vectors at the relay gives the best performance. Then, we propose a Max-Min antenna selection with binary network coding scheme that can approach this performance with only partial channel state information.

## I. INTRODUCTION

**B**I-DIRECTIONAL communications via a relay is often encountered in intra-cell, intra-hotspot, or more recently intra-picocell communication.

Traditionally, bi-directional relaying requires 4 channel uses (or 4 steps) as communication in each direction requires 2 channel uses, as shown in Fig. 1. However, the efficiency of the scheme can be improved by introducing network coding [?] at the relay node. The relay node may perform either analog network coding (ANC) [?], [?], [?] or binary network coding (BNC) [?], [?], while the destination nodes perform self interference cancellation, to reduce the channel uses of bi-directional relaying to 3 channel uses (or 3 steps). In this paper, we consider a 2-step approach by using two or more antennas at the relay, so that the relay node is able to jointly detect both messages sent from the two source nodes. The two messages are then combined using either binary eXclusive-OR (XOR) [?], [?] or analog beamforming. This two-step procedure is illustrated in Fig. 2.

In this paper, we study two different relaying schemes, ANC-based Transmit Beamforming (TB) and BNC-based Space Time Block Coding (STBC-BNC), and propose a novel BNC-based Max-Min Antenna Selection (Max-Min AS-BNC) scheme. In the Max-Min AS-BNC scheme, we select an antenna that has

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the largest channel gain among the worst channels from each antenna to either nodes A or B. We analyze the diversity gain and symbol error probability for the various multi-antenna relaying schemes to get an insight into their relative system performance and to compare their implementation requirements such as the need for downlink ( $2^{nd}$  step in Fig. 2) channel state information (CSI), *i.e.* phase and amplitude of the channels between each relay antennas and each node (A and B). It is shown that Max-Min AS-BNC achieves full diversity order with the need of coarse downlink CSI, *i.e.* the relative amplitude of the downlink channel, and approaches the performance of Transmit Beamforming (TB) based on maximal ratio AF transmission which requires exact CSI, and performs better than Space Time Block Coding with Binary Network Coding (STBC-BNC) which requires no CSI.

In this paper, bold lower case and upper case letters denote vectors and matrices, respectively;  $[\cdot]^T$  the transpose of a vector or a matrix;  $|\cdot|$ , the absolute value of a real number;  $\|\cdot\|$ , the norm of a vector and  $Q(x)$ , the  $Q$ -function of  $x$  which is the probability that a standard normal random variable will obtain a value larger than  $x$ .

The paper is organized as follows. In the next section, we present the system model for the different two-step two-way relaying schemes. In Section III, we analyze the downlink channel for different relaying schemes, in terms of symbol error probability and diversity order. We present the numerical results and compare them with our analytical results in Section IV. We then conclude our paper in Section V.

## II. SYSTEM MODEL

We consider a network comprising of a pair of single-antenna nodes (nodes A and B that would like to exchange information) and one  $N$ -antenna relay node (node R). All nodes are half-duplex and there is no direct link between nodes A and B. In the first transmission interval, nodes A and B transmit  $s_A$  and  $s_B$  respectively to the relay R simultaneously and on the same frequency. The multi-antenna relay R then uses maximum likelihood (ML) detection to detect them jointly.

We assume that the probability distribution of all the uplink channels and the downlink channels are the same, and the noise at both nodes A and B is complex zero-mean additive white Gaussian with variance  $\sigma_S^2$ . We will analyze the performance of the system for the information flow from node B to node A, where the results hold for the information flow from node A to node B by assuming that the channels have the same distribution statistic.

In the first time slot (uplink), the relay R receives

$$\mathbf{y}_R = \mathbf{H}_{up}\mathbf{s} + \mathbf{n}_R \quad (1)$$

where  $\mathbf{s} = (s_A \ s_B)^T$ ,  $\mathbf{h}_{AR} = (h_{A1} \dots h_{AN})^T$ ,  $\mathbf{h}_{BR} = (h_{B1} \dots h_{BN})^T$ ,  $\mathbf{H}_{up} = (\mathbf{h}_{AR} \ \mathbf{h}_{BR})$ , and  $\mathbf{n}_R$  is a circularly symmetric complex Gaussian noise vector  $\sim \mathcal{CN}(\mathbf{0}_N, \sigma_R^2 \mathbf{I}_N)$ .  $h_{kj}$  is a Rayleigh flat fading

channel with power mean  $E[|h_{kj}|^2] = \sigma_0^2$  between node  $k$  and the  $j^{th}$  relay antenna where  $k = A$  or  $B$  and  $j \in \{1, \dots, N\}$ .

We assume that  $\mathbf{H}_{up}$  is known at the relay. So, the relay performs maximum-likelihood decoding on the received vector to obtain

$$\hat{\mathbf{s}} = (\hat{s}_A \ \hat{s}_B)^T = \arg \min_{\mathbf{s}} \|\mathbf{y}_R - \mathbf{H}_{up}\mathbf{s}\|^2 \quad (2)$$

where  $\hat{s}_A$  and  $\hat{s}_B$  are the estimation of the transmitted symbols by nodes  $A$  and  $B$ .

In the second time slot (downlink), the relay uses different multi-antenna precoding schemes to broadcast a message, that is formed by  $\hat{s}_A$  and  $\hat{s}_B$ , to both nodes  $A$  and  $B$ . If the downlink scheme is based on binary network coding (BNC), the relay performs bit-wise XOR operation on the estimated symbols  $\hat{s}_A$  and  $\hat{s}_B$  to obtain the network coded message

$$s_{XOR} = \hat{s}_A \oplus \hat{s}_B.$$

Then, the relay broadcasts  $\mathbf{s}_R = f(s_{XOR})$ , which is a function of  $s_{XOR}$ . If the downlink scheme is based on analog network coding (ANC),  $\mathbf{s}_R$  is a linear combination of the symbols  $\hat{s}_A$  and  $\hat{s}_B$ ,  $\mathbf{s}_R = f(\hat{s}_A, \hat{s}_B)$ . In both cases,  $\mathbf{s}_R$  is broadcasted by the relay to both nodes  $A$  and  $B$  in the second time slot, as shown in Fig. 2.

We denote the downlink channel vectors from the relay node to nodes  $A$  and  $B$  by  $\mathbf{h}_{RA} = (h_{1A} \dots h_{NA})^T$  and  $\mathbf{h}_{RB} = (h_{1B} \dots h_{NB})^T$  respectively. Without loss of generality,  $h_{jk}$  is a Rayleigh flat fading channel with the same mean power as the uplink channels, *i.e.*  $E[|h_{jk}|^2] = \sigma_0^2$  between the  $j^{th}$  relay antenna and node  $k$  where  $k = A$  or  $B$  and  $j \in \{1, \dots, N\}$ . The average SNR at the relay node and at node  $A$  or  $B$  are  $\zeta_R = \frac{\mathcal{P}}{\sigma_R^2}$  and  $\zeta_S = \frac{\mathcal{P}}{\sigma_S^2}$  respectively, where  $\mathcal{P} = E[|s_A|^2] = E[|s_B|^2] = E[|\mathbf{s}_R|^2]$ .

Node  $A$  receives

$$y_A = \mathbf{h}_{RA}^T \mathbf{s}_R + n_A \quad (3)$$

If the relaying scheme is based on BNC, after estimating  $s_{XOR}$ , node  $A$  performs bit-wise XOR operation on the estimated  $s_{XOR}$  and  $s_A$  to estimate  $s_B$ . If the relaying scheme is based on ANC, after canceling off the contribution of  $s_A$  from the received signal, node  $A$  estimates  $s_B$ .

We consider three different relaying schemes which require different amount of downlink channel state information (CSI) at the relay:

- No CSI: STBC-BNC relaying scheme,
- Coarse CSI: Antenna selection relaying scheme (*i.e.* Max-Min AS-BNC),
- Exact CSI: Transmit beamforming (TB) relaying scheme (based on ANC).

In the exact CSI case, the relay knows the amplitude and phase of the downlink channels. In the coarse CSI case, the relay just needs to know the relative amplitude of the downlink channels. Although CSI can be



inferred from the uplink in TDD system, this is usually not feasible due to non-symmetrical impairments in the transmit and receive circuitry. Therefore, the proposed Max-Min antenna selection that requires only coarse CSI is more practical than the transmit beamforming that requires perfect CSI.

We will show later that Max-Min antenna selection with BNC can achieve full diversity, and it approaches the performance of TB relaying scheme asymptotically. In addition to the fact that Max-Min AS-BNC just needs coarse CSI, another advantage of using Max-Min AS-BNC relaying scheme over TB is that in a distributed network of multiple-antenna relays, implementing Max-Min AS-BNC is much easier than implementing TB. To implement Max-Min AS-BNC, a method based on time is selected: each relay will start its own timer with an initial value, inversely proportional to  $g_k$ . The relay with the least initial value or the most  $g_k$  gets zero earlier than the other relays [?]. In this case to implement TB scheme we need much more complicated algorithm to select one or two relay(s) to transmit  $\mathbf{s}_R$  which results in the most instantaneous SNR at nodes A and B.

### III. DOWNLINK ANALYSIS

In the following, we derive the diversity order and symbol error probability of the different downlink channel schemes and hence of the whole system. We denote an exponential random variable  $D$  with parameter  $\lambda$  as one with  $E[D] = 1/\lambda$ ; a Gamma-distributed random variable  $T$  with parameters  $(\Omega, \theta)$  as one with  $E[T] = \Omega\theta$  and  $var(T) = \Omega\theta^2$ . Also, a random variable  $Z$  which is complex Gaussian-distributed with parameters  $(\mu, \sigma^2)$  has  $E[Z] = \mu$  and  $var(Z) = \sigma^2$ .

We assume both nodes A and B have sufficient downlink channel knowledge to perform decoding. We employ M-PSK constellation and coherent detection at the receiver, so according to [?], [?], [?], [?], we can write the symbol error probability (SEP) for the downlink channels as

$$P_d = E_\gamma[P_{dA}] = 1/\pi \int_0^{\pi-\pi/M} \psi_\gamma(g_{MPSK}/\sin^2 \theta) d\theta, \quad (4)$$

where  $P_{dA}$  is the SEP of the downlink channels at node A,  $g_{MPSK} = \sin^2(\pi/M)$ ,  $\gamma$  is the instantaneous receive SNR and  $\psi_\gamma(t)$  is the moment generating function (MGF) of  $\gamma$ . Note that the proposed schemes can be applied to any modulation. We use the M-PSK to demonstrate how the analysis can be done, and the extension to other constellations is straightforward.

According to [?], the diversity order of the downlink channel can be found by

$$\lim_{\zeta_S \rightarrow \infty} -\frac{\log(\psi_\gamma(g_{MPSK}))}{\log(\zeta_S)}. \quad (5)$$

#### A. Max-Min Antenna Selection with Binary Network Coding (Max-Min AS-BNC)

In the Max-Min AS-BNC scheme, we select just one antenna of the relay node to send  $s_{XOR}$ , at all times. The process of selection is done in two steps. First, we select the worst downlink channel  $h_{ik_i}$  for

every antenna  $i \in \{1, \dots, N\}$ , *i.e.*

$$k_i = \arg \min_r |h_{ir}|, \quad r \in \{A, B\} \quad (6)$$

Then, the relay antenna  $j$  with the best downlink channel among those in (6) is selected, *i.e.*

$$j = \arg \max_i |h_{ik_i}|, \quad i \in \{1, \dots, N\}. \quad (7)$$

This antenna  $j$  at the relay node is then used to broadcast  $s_{XOR}$  to nodes A and B. So, node A receives

$$y_A = h_{jA}s_{XOR} + n_A. \quad (8)$$

Hence, the instantaneous SNR,  $\gamma$ , at node A is

$$\gamma = |h_{jA}|^2 \zeta_S. \quad (9)$$

In this relaying scheme, there are two states. State 1 is when the channel magnitude from the selected antenna at the relay node to node A is less than to node B, *i.e.*  $|h_{jA}| < |h_{jB}|$ , or

$$\min\{|h_{jA}|, |h_{jB}|\} = |h_{jA}|, \quad (10)$$

and State 2 is when the channel magnitude from the selected antenna at the relay node to node A is more than to node B, *i.e.*  $|h_{jA}| > |h_{jB}|$ , or

$$\min\{|h_{jA}|, |h_{jB}|\} = |h_{jB}|. \quad (11)$$

Hence, the SEP at node A is

$$\text{SEP} = \text{SEP}_1 P_1 + \text{SEP}_2 P_2, \quad (12)$$

where  $\text{SEP}_i$  is the SEP conditioned on State  $i$  and  $P_i$  is the probability of occurrence of State  $i$ . Since the channel gain in State 1 is less than the channel gain in State 2, the symbol error probability of State 1 will be higher than the symbol error probability of State 2, *i.e.*  $\text{SEP}_1 > \text{SEP}_2$ . Since all channels are i.i.d,  $P_1 = P_2 = 1/2$ , the overall SEP will be upper-bounded by  $\text{SEP}_1$  and lower-bounded by  $\text{SEP}_1/2$ , *i.e.*

$$\frac{\text{SEP}_1}{2} \leq \text{SEP} \leq \text{SEP}_1. \quad (13)$$

To use the MGF to work out the diversity order and SEP of the downlink channel, we need to know the PDF of  $|h_{jA}|^2$  in State 1.

From [?], the cumulative density function (CDF) of  $|h_{ik_i}|^2$  can be shown to be

$$F(|h_{ik_i}|^2) = 1 - \exp(-2|h_{ik_i}|^2/\sigma_0^2). \quad (14)$$

Hence, the PDF of  $|h_{jA}|^2$  in State 1 is

$$f(|h_{jA}|^2) = \frac{2N}{\sigma_0^2} \exp(-2|h_{jA}|^2/\sigma_0^2) (1 - \exp(-2|h_{jA}|^2/\sigma_0^2))^{N-1}. \quad (15)$$

Using binomial expansion, we have

$$(1 - \exp(-2|h_{jA}|^2/\sigma_0^2))^{N-1} = \sum_{k=0}^{N-1} \binom{N-1}{k} (-1)^k \exp(-2k|h_{jA}|^2/\sigma_0^2). \quad (16)$$

The MGF of the instantaneous SNR is thus

$$\begin{aligned} \psi_\gamma(t) &= E[\exp(-t\gamma)] \\ &= \int_0^\infty \exp(-t|h_{jA}|^2\zeta_S) f(|h_{jA}|^2) d|h_{jA}|^2. \end{aligned} \quad (17)$$

Substituting (16) into (15) and then using (17), the MGF of the instantaneous SNR in the Max-Min AS-BNC scheme in State 1 is

$$\begin{aligned} \psi_\gamma(t) &= \sum_{k=0}^{N-1} \binom{N-1}{k} \frac{2N(-1)^k}{\sigma_0^2} \int_0^\infty \exp(-|h_{jA}|^2(t\zeta_S + 2(k+1)/\sigma_0^2)) d|h_{jA}|^2 \\ &= \sum_{k=0}^{N-1} \binom{N-1}{k} \frac{N(-1)^k}{1 + k + t\sigma_0^2\zeta_S/2} \end{aligned} \quad (18)$$

$$= \frac{N!}{\prod_{k=0}^{N-1} (k+1 + t\sigma_0^2\zeta_S/2)}. \quad (19)$$

The equality between (18) and (19) can be easily shown using partial fraction decomposition [?].

According to (5), the diversity order of the downlink channel of the Max-Min AS-BNC scheme in State 1 is

$$\begin{aligned} G_d &= \lim_{\zeta_S \rightarrow \infty} -\frac{\log N!}{\log \zeta_S} + \lim_{\zeta_S \rightarrow \infty} \frac{\sum_{k=0}^{N-1} \log(k+1 + g_{MPSK}\sigma_0^2\zeta_S/2)}{\log \zeta_S} \\ &= 0 + N = N. \end{aligned} \quad (20)$$

Following the inequalities in (13), we conclude that the Max-Min AS-BNC relaying scheme is able to achieve the full diversity order of  $N$  for the downlink channel and also for the whole system.

Similarly, using (19), the SEP of the downlink channel can be lower-bounded as

$$P_d^{Max-Min} = E_h[P_{eA}] \geq \frac{1}{2\pi} \int_0^{\pi-\pi/M} \psi_\gamma\left(\frac{g_{MPSK}}{\sin^2 \theta}\right) d\theta. \quad (21)$$

### B. Transmit Beamforming (TB)

In the TB downlink relaying scheme, the detected uplink data  $\hat{s}_A$  and  $\hat{s}_B$  are precoded using analog transmission weight vectors  $\mathbf{v}_B$  and  $\mathbf{v}_A$  respectively based on the principles of maximum ratio transmission (MRT) [?]

$$\mathbf{v}_A = \frac{\mathbf{h}_{RA}^*}{\|\mathbf{h}_{RA}\|}, \mathbf{v}_B = \frac{\mathbf{h}_{RB}^*}{\|\mathbf{h}_{RB}\|}. \quad (22)$$

The relay R then sends

$$\mathbf{s}_R = \frac{\mathbf{v}_B \hat{s}_A + \mathbf{v}_A \hat{s}_B}{\sqrt{2}}. \quad (23)$$

where  $\sqrt{2}$  is to normalize the transmission power at the relay node. The receiver output at node A, after subtracting  $\mathbf{h}_{RA}^T \mathbf{v}_B \hat{s}_A$  from the received vector is

$$y_A = \frac{\mathbf{h}_{RA}^T \mathbf{v}_A \hat{s}_B}{\sqrt{2}} + n_A = \frac{\|\mathbf{h}_{RA}\| \hat{s}_B}{\sqrt{2}} + n_A. \quad (24)$$

The instantaneous SNR ( $\gamma$ ) at node A will be

$$\gamma = \frac{\|\mathbf{h}_{RA}\|^2 \zeta_S}{2}. \quad (25)$$

Since we assume perfect cancellation at the receiver, the downlink channel is equivalent to a point-to-point MISO channel. The optimal precoding scheme for point-to-point transmission is MRT. This explains our choice of (22) as the TB weight vectors. Since TB linearly sums  $\hat{s}_A$  and  $\hat{s}_B$ , it is a form of ANC (analog network coding). From [?], the diversity order of MRT is  $N$ , therefore the diversity order of the downlink channel and the entire bi-directional relaying system is  $N$ .

Since  $\|\mathbf{h}_{RA}\|^2 = \sum_{i=1}^N |h_{iA}|^2$  is the sum of  $N$  independent exponentially distributed random variables each with  $\lambda = 1/\sigma_0^2$ ,  $\|\mathbf{h}_{RA}\|^2$  is a Gamma random variable with parameters  $(N, \sigma_0^2)$ . Hence, the MGF of  $\gamma$  is

$$\psi_\gamma(t) = \left( \frac{\lambda}{\lambda + t\zeta_S/2} \right)^N = \left( \frac{1}{1 + t\sigma_0^2\zeta_S/2} \right)^N, \quad (26)$$

Using (26) and (4), the SEP can be written as

$$P_d^{TB} = E_h[P_{dA}] = \frac{1}{\pi} \int_0^{\pi-\pi/M} \psi_\gamma \left( \frac{g_{MPSK}}{\sin^2 \theta} \right) d\theta. \quad (27)$$

### C. Space-Time Block Coding with Binary Network Coding (STBC-BNC)

For the STBC-BNC scheme, at the relay node, binary network coded symbols at times  $n, \dots, n+N$  are buffered and encoded using STBC to produce  $\mathbf{s}_R$ . Alamouti coding is an orthogonal STBC for two transmit antennas, which achieves full diversity order without sacrificing transmission rate. In the Alamouti-BNC scheme, the downlink signal  $\mathbf{s}_R$  is

$$\mathbf{s}_R = \begin{pmatrix} s_{XOR}(n) & -s_{XOR}^*(n+1) \\ s_{XOR}(n+1) & s_{XOR}^*(n) \end{pmatrix}.$$

Correspondingly, node A receives

$$\begin{pmatrix} y_A(n) & y_A(n+1) \end{pmatrix} = \frac{\mathbf{h}_{RA}^T \mathbf{s}_R}{\sqrt{2}} + n_A. \quad (28)$$

In general, by using orthogonal STBC at an  $N$ -antenna relay, it is easy to show that the instantaneous SNR ( $\gamma$ ) received at the destination node is

$$\gamma = \frac{\|\mathbf{h}_{RA}\|^2 \zeta_S}{N}, \quad (29)$$

which is the same as (25) for the TB relaying scheme except for the denominator of  $N$  instead of 2. Hence, we conclude that the diversity order of the downlink channel and also the whole system, is  $N$  for the STBC-BNC relaying scheme.

For the same reason, the MGF of  $\gamma$  is the same as (26) with  $\zeta_S$  changed to  $\zeta_S/N$

$$\psi_\gamma(t) = \left( \frac{1/\sigma_0^2}{1/\sigma_0^2 + t\zeta_S/N} \right)^N. \quad (30)$$

Using (30) and (4), the SEP can be written as

$$P_d^{\text{STBC-BNC}} = E_h[P_{dA}] = \frac{1}{\pi} \int_0^{\pi-\pi/M} \psi_\gamma \left( \frac{g_{\text{MPSK}}}{\sin^2 \theta} \right) d\theta. \quad (31)$$

#### IV. SIMULATION AND DISCUSSION

##### A. Diversity and SEP Comparison

We approximate the overall performance by assuming that overall bi-directional communication will be erroneous if an error event occurs in either the uplink or the downlink. Hence,

$$P_{\text{tot}} \approx P_u + P_d(1 - P_u), \quad (32)$$

where  $P_u$ ,  $P_d$  and  $P_{\text{tot}}$  are SEP of the uplink channel, downlink channel and the whole system respectively. Since  $P_u P_d \ll 1$ , (32) can be simplified to

$$P_{\text{tot}} \approx P_u + P_d. \quad (33)$$

The union bound property [?] is used to compute an upper bound on  $P_u$ ,

$$P_u \leq \sum_{S \neq S_0} Q \left( \sqrt{\frac{\|(\mathbf{S}_0 - \mathbf{S})\mathbf{h}\|^2}{2\sigma_R^2}} \right), \quad (34)$$

while the SEP derived for the different downlink schemes in (21), (27) and (31) will be used as  $P_d$  in (33).

In Fig. 3, we plot the analytical and simulated SEP of the ANC and BNC-based relaying schemes. Close agreement between the analytical result and the simulation result, especially in the high SNR region, is

observed. Also, TB has an advantage of 0.5 dB over the BNC-based schemes. Interestingly, the Max-Min AS-BNC scheme and Alamouti-BNC scheme have very close performance at all SNRs.

In order to verify our analysis of the diversity order of Max-Min AS-BNC, we show the simulation results of the downlink portion with various antenna configuration in Fig. 4. From the figure, we can corroborate that the scheme achieves full downlink diversity order. Since the uplink performs ML decoding, we may conclude that the scheme achieves full end-to-end diversity order too. Additionally, we compare the Max-Min AS-BNC and STBC-BNC schemes by simulation for four-antenna relay in Fig. 5. It shows that the Max-Min AS-BNC scheme has a coding gain over the other. This advantage will be analyzed in the next section.

### B. Asymptotic Analysis

We analyze the asymptotic SEP ratio of downlink relaying scheme A relative to scheme B as:  $\frac{SEP_A}{SEP_B} \big|_{SNR \rightarrow \infty}$ . If this ratio is less than one, the relaying scheme A is deemed to have SEP performance gain over the relaying scheme B asymptotically.

We begin with the SEP ratio of TB over Max-Min AS-BNC

$$\begin{aligned}
 \frac{SEP_{TB}}{SEP_{Max-Min \text{ AS-BNC}}} \big|_{SNR \rightarrow \infty} &= \lim_{\zeta_S \rightarrow \infty} \frac{\int_0^{\frac{\pi}{M}} \frac{2 \sin^2 \theta}{(\sin^2 \theta + g_{MPSK} \sigma_0^2 \zeta_S / 2)^N} d\theta}{\int_0^{\frac{\pi}{M}} \frac{N! \sin^2 \theta}{\prod_{k=0}^{N-1} (\sin^2 \theta (k+1) + g_{MPSK} \sigma_0^2 \zeta_S / 2)} d\theta} \\
 &= \lim_{\zeta_S \rightarrow \infty} \frac{\int_0^{\frac{\pi}{M}} \frac{2 \sin^2 \theta}{(g_{MPSK} \sigma_0^2 \zeta_S / 2)^N} d\theta}{\int_0^{\frac{\pi}{M}} \frac{N! \sin^2 \theta}{\prod_{k=0}^{N-1} (g_{MPSK} \sigma_0^2 \zeta_S / 2)} d\theta} \\
 &= \lim_{\zeta_S \rightarrow \infty} \frac{(g_{MPSK} \sigma_0^2 \zeta_S / 2)^N \int_0^{\frac{\pi}{M}} 2 \sin^2 \theta d\theta}{(g_{MPSK} \sigma_0^2 \zeta_S / 2)^N N! \int_0^{\frac{\pi}{M}} \sin^2 \theta d\theta} \\
 &= \frac{2}{N!}.
 \end{aligned} \tag{35}$$

Next, the SEP ratio of STBC-BNC over Max-Min AS-BNC is

$$\begin{aligned}
 \frac{SEP_{STBC-BNC}}{SEP_{Max-Min \text{ AS-BNC}}} \big|_{SNR \rightarrow \infty} &= \lim_{\zeta_S \rightarrow \infty} \frac{\int_0^{\frac{\pi}{M}} \frac{2 \sin^2 \theta}{(\sin^2 \theta + g_{MPSK} \sigma_0^2 \zeta_S / N)^N} d\theta}{\int_0^{\frac{\pi}{M}} \frac{N! \sin^2 \theta}{\prod_{k=0}^{N-1} (\sin^2 \theta (k+1) + g_{MPSK} \sigma_0^2 \zeta_S / 2)} d\theta} \\
 &= \lim_{\zeta_S \rightarrow \infty} \frac{\int_0^{\frac{\pi}{M}} \frac{2 \sin^2 \theta}{(g_{MPSK} \sigma_0^2 \zeta_S / N)^N} d\theta}{\int_0^{\frac{\pi}{M}} \frac{N! \sin^2 \theta}{\prod_{k=0}^{N-1} (g_{MPSK} \sigma_0^2 \zeta_S / 2)} d\theta} \\
 &= \lim_{\zeta_S \rightarrow \infty} \frac{(g_{MPSK} \sigma_0^2 \zeta_S / N)^N \int_0^{\frac{\pi}{M}} 2 \sin^2 \theta d\theta}{(g_{MPSK} \sigma_0^2 \zeta_S / 2)^N N! \int_0^{\frac{\pi}{M}} \sin^2 \theta d\theta} \\
 &= \frac{N^N}{2^{N-1} N!}.
 \end{aligned} \tag{36}$$

(35) and (36) suggest that with two antennas at the relay, all the relaying schemes have the same downlink SEP performance, while with more than two antennas at the relay, TB achieves the best

downlink SEP performance, followed by Max-Min AS-BNC, then STBC-BNC. These analysis explain the observations in Fig. 3 and Fig. 5 respectively. Note that, although we use the lower bound of the SEP of Max-Min AS-BNC in the SEP ratio derivation, the simulation results in Fig. 3 have shown that the lower bound is very tight especially in the high SNR region.

## V. CONCLUSION

In this paper, we present and analyze three different multi-antenna relaying schemes with binary or analog network coding at the relay that achieve bi-directional wireless relay communication in two steps. The relaying scheme with analog network coding employs Transmit Beamforming (TB) with maximal ratio transmission. The relaying schemes with binary network coding employ Max-Min Antenna Selection (Max-Min AS-BNC) and Space-Time Block Coding (STBC-BNC). We derive the diversity order and symbol error probability formulas of these two-step bi-directional relaying schemes considering two single-antenna nodes connected via a multi-antenna relay.

We show that while all the relaying schemes presented achieve full diversity order, different schemes allow for different trade-offs between complexity and performance. Specifically, TB performs best in terms of SEP performance, but it needs exact downlink CSI at the relay. Max-Min AS-BNC comes next and is able to tolerate coarse CSI and can be implemented in a distributed multiple-relay system, followed by STBC-BNC which does not require CSI. Asymptotic SEP ratios are also derived to help evaluate the relative performance of these full-diversity two-way relaying schemes when the SNR is high.

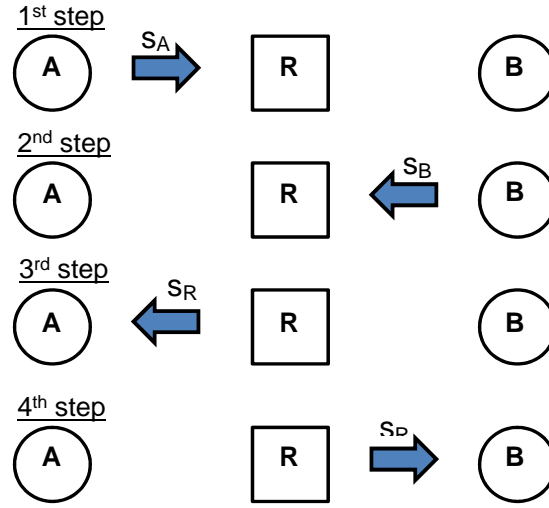


Fig. 1. Conventional four-step bi-directional relaying communication.

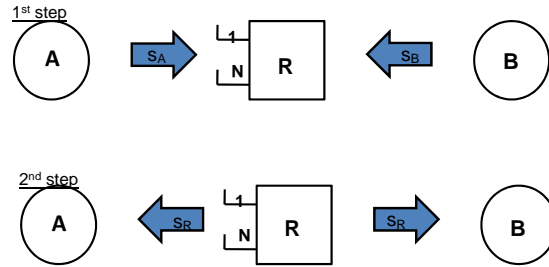


Fig. 2. Two-Step bi-directional communication with multi-antenna relay. Relay R broadcasts precoded versions of  $s_R = \hat{s}_A \oplus \hat{s}_B$  or linear combination of the symbols  $\hat{s}_A$  and  $\hat{s}_B$  to nodes A and B in the second time slot.

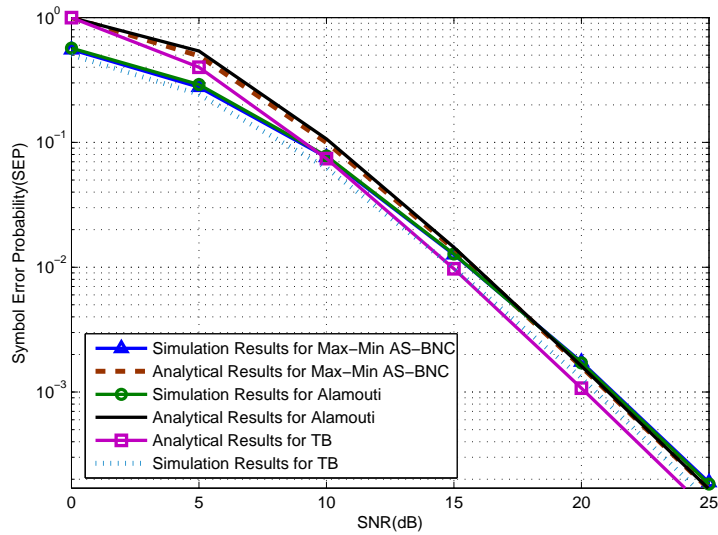


Fig. 3. Simulation and Analytical SEP of two-way relaying using QPSK with TB, Max-Min AS-BNC and Alamouti-BNC.



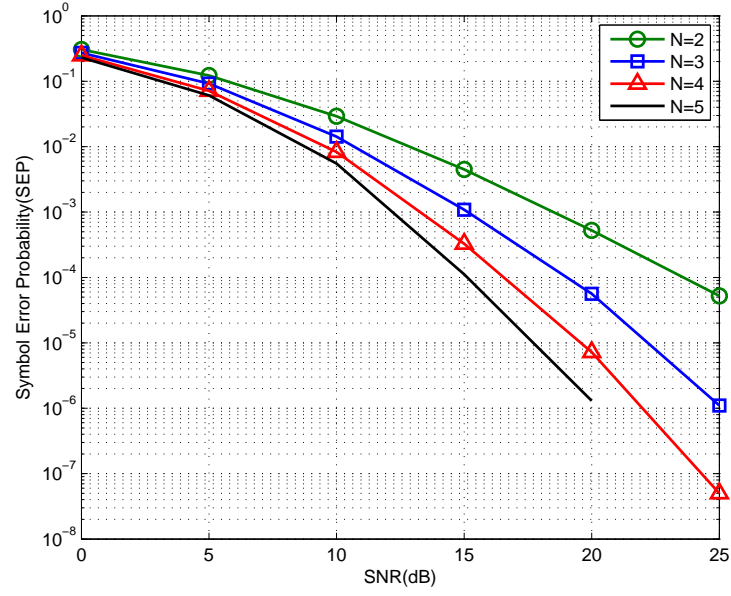


Fig. 4. Simulation results of downlink (2nd step) of Max-Min AS-BNC with different relay antenna number  $N$ .

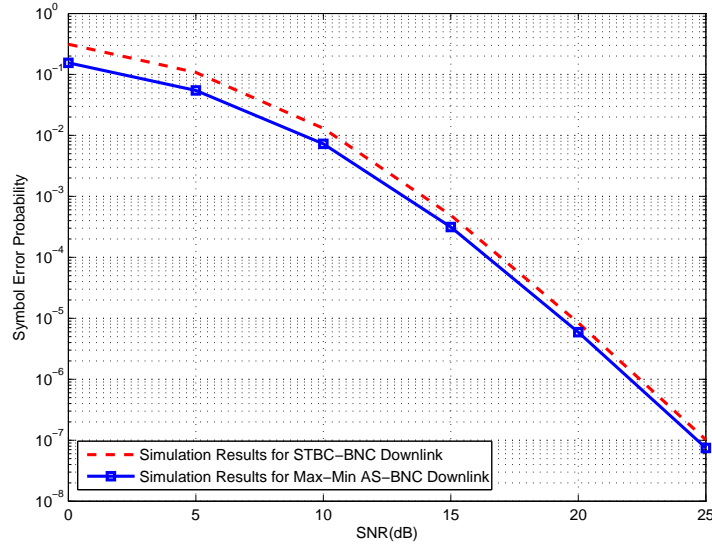


Fig. 5. Analytical SEP of two-way relaying using QPSK with Max-Min AS-BNC and STBC-BNC for four-antenna relay.